Department of Basic Science Level: 1 Examiner: Dr. Mohamed Eid Time allowed: 3 hours	معهد الأهرامات العالى للهندسة و التكنولوجيا	Prep. Year: Final Exam Course: Mathematics 2 Course Code: BAS 013 B Date: May, 2016

Answer all questions No. of questions: 5 Total Mark: 70

Question 1

Find **y`** from the following: 18

(a) $y = 3 + 3^x + 3^{x^2}$ (b) $y = x^3 . \cosh x^2$

(c) $\mathrm{y = \ln x . \tanh x}$ (d) $y = \tan^{-1} x + \sinh^{-1}\mathrm{x}$

(e) $\mathrm{y = t.\ln t\ ,\ \ x = t.e^t}$ (f) $\mathrm{y^2 = x^y + \ln(x + y)}$

Question 2

18

Find the following integrals:

(a) $\int (x^2 + \frac{1}{x^2} + 2^x)dx$ (b) $\int (\frac{x}{1+x^2} + \frac{x}{\sqrt{1+x^2}})dx$

(c) $\int (x - \sqrt{x})^2\, dx$ (d) $\int (\frac{1}{3} + \frac{1}{\mathrm{x}} - \frac{3}{\mathrm{x+1}})\mathrm{dx}$

(e) $\int e^x (3 + e^x)^7\, dx$ (f) $\int x \ln x\, dx$

(g) $\int \mathrm{\sin x . \cos 4x\, dx}$ (h) $\int \mathrm{\cos^4 x\, dx}$ (i) $\int \frac{x}{x^2-3x-4} dx$

Question 3

(a)Find the area of the region between the curve $y = x^2 - 1$, x-axis, x in [0, 3]. 4

(b)If the region between the curve $y = 1 + e^x$, x-axis, x in [1, 2] is rotated about (i) x-axis (ii)y-axis. 8

Find the volume of the generated solids V_x , V_y.

(c)Find the length of the curve $y = \frac{x^3}{12} + \frac{1}{x}$, x in [2, 3]. 4

Question 4

(a)State the definition of the plane. 2

(b)Find the angle between the planes : 3

x – 2y + 2z + 5 = 0 , 2x + y + 2z – 1= 0.

(c)Write the equation of the plane that passes through the 5

points: (2, 1, 3), (–1, 0, 4), (1, 0, –1).	
<u>Question 5</u>	
(a)State the definition of the sphere.	2
(b)Write the equation of the plane that passes through the point (3, –1, 2) and its normal vector $\overline{\mathrm{N}} = \mathrm{i} - 3\mathrm{j} + 2\mathrm{k}$.	2
(c)Write the name of each surface: (i) $x^2 + y^2 + z^2 - x + 3z = 0$ (ii) $z^2 - x^2 - 3y^2 = 0$ (iii) $2y^2 + z^2 = 4$ (iv) $x^2 + z^2 = 2y^2$	4

Good Luck *Dr. Mohamed Eid*

<u>*Answer*</u>

<u>Question 1</u>

(a) $y` = 0 + 3^x . \ln 3 + 3^{x^2} . \ln 3 \, . \, 2x$

(b) $y1 = x^3 . \sinh x^2 \, . \, 2x + 3x^2 \cosh x^2$

(c) $y` = \frac{1}{x} . \tanh x + \ln x \, . \, \mathrm{sech}^2 x$

(d) $y` = \tan^{-1} x + \sinh^{-1} x = \frac{1}{1+x^2} + \frac{1}{\sqrt{1+x^2}}$

(e) $y` = \frac{1+\ln t}{e^t + te^t}$

(f) $y^2 = e^{y \ln x} + \ln(x + y)$

Then $2yy` = e^{y \ln x}[\frac{y}{x} + y` \ln x] + \frac{1+y`}{x+y}$

Then $y`(2y - e^{y \ln x} \ln x - \frac{1}{x+y}) = e^{y \ln x} \frac{y}{x} + \frac{1}{x+y}$

Then $y` = \frac{e^{y \ln x} \frac{y}{x} + \frac{1}{x+y}}{2y - e^{y \ln x} \ln x - \frac{1}{x+y}}$

--18-marks

Question 2

(a) $\int (x^2 + \frac{1}{x^2} + 2^x) dx = \frac{x^3}{3} - \frac{1}{x} + \frac{2^x}{\ln 2} + c$

(b) $\int (\frac{x}{1+x^2} + \frac{x}{\sqrt{1+x^2}}) dx = \frac{1}{2}\ln(1 + x^2) + \sqrt{1 + x^2} + c$

(c) $\int (x - \sqrt{x})^2 \, dx = \int (x^2 + x - 2x^{3/2}) dx = \frac{x^3}{3} + \frac{x^2}{2} + \frac{4}{5}x^{5/2} + c$

(d) $\int (\frac{1}{3} + \frac{1}{x} - \frac{3}{x+1}) dx = \frac{1}{3}x + \ln x - 3\ln(x + 1) + c$

(e) $\int e^x (3 + e^x)^7 \, dx = \frac{1}{8} \, (3 + e^x)^8 + c$

(f) $\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x \, dx = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$

(g) $\int \sin x . \cos 4x \, dx = \frac{1}{2}\int (-\sin 3x + \sin 5x) \, dx$

$$= \frac{1}{2}\left(\frac{1}{3}\cos 3x - \frac{1}{5}\cos 5x\right) + c$$

(h) $\int \cos^4 x \, dx = \frac{1}{4}\int (1 + \cos 2x)^2 \, dx$

$$= \frac{1}{4}\int (1 + 2\cos 2x + \frac{1}{2}(1 + \cos 4x)) \, dx$$

$$= \frac{1}{4}\left(\frac{3}{2}x + \sin 2x + \frac{1}{8}\sin 4x\right) + c$$

(i)By partial fractions, $\frac{2x-1}{x^2-3x-4} = \frac{A}{x-4} + \frac{B}{x+1} = \frac{A(x+1)+B(x-4)}{x^2-3x-4}$

Then $x = A(x + 1) + B(x - 4)$

Putting x = 4, then A = 4/5.

Putting x = – 1, then B = 1/5.

Then $I = \int (\frac{\frac{4}{5}}{x-4} + \frac{\frac{1}{5}}{x+1}) \, dx = \frac{4}{5}\ln(x - 4) + \frac{1}{5}\ln(x + 1) + c$

--18-Marks

Question 3

(a)Solving the equation : $x^2 - 1 = 0$, we get x = 1, x = –1.

We see that x = 1 lies inside the interval [0, 3].

Then, the area is :

$$A = \int_0^1 (x^2 - 1)\, dx + \int_1^3 (x^2 - 1)\, dx = \left|-\frac{2}{3}\right| + \frac{20}{3} = \frac{22}{3}$$

---4-Marks

(b)The volumes are :

$$V_x = \pi \int_1^2 (1 + e^x)^2\, dx = 33.95\pi, \quad V_y = 2\pi \int_1^2 x(1 + e^x) \quad dx = 17.78\pi$$

---8-Marks

(c)The length of the curve is :

$$L = \int_2^3 \sqrt{1 + (y`)^2}\, dx = \int_2^3 (\frac{1}{4}x^2 + \frac{1}{x^2})\, dx = \frac{7}{4}$$

---4-Marks

Question 4

(a)State the definition of the plane.

---2-Marks

(b)The normal vector of the first plane is : $\overline{N_1} = i - 2j + 2k$

The normal vector of the first plane is : $\overline{N_2} = 2i + j + 2k$

The angle between the planes is :

$$\cos\theta = \frac{\overline{N_1}.\overline{N_2}}{|\overline{N_1}|.|\overline{N_2}|} = \frac{2-2+4}{3x3} = \frac{4}{9}$$

---3-Marks

(c)The vector $\overline{P_1P_2} = -3i - j + k$ and the vector $\overline{P_1P_3} = -i - j - 4k$.

The normal vector of the required plane is :

$$\overline{P_1P_2} \times \overline{P_1P_3} = \begin{vmatrix} i & j & k \\ -3 & -1 & 1 \\ -1 & -1 & -4 \end{vmatrix} = 5i - 13j + 2k$$

Then the required plane is : $5(x - 2) - 13(y - 1) + 2(z - 3) = 0$.

Or $5x - 13y + 2z - 3 = 0$.

---5-Marks

Question 5

(a)Definition of the sphere.

---2-marks

(b)The equation of plane is :

$(x - 3) - 3(y + 1) + 2(z - 2) = 0$ Or $x - 3y + 2z - 10 = 0$

---2-marks

(c)(i) $x^2 + y^2 + z^2 - x + 3z = 0$ Sphere

(ii) $z^2 - x^2 - 3y^2 = 0$ Cone with z – axis

(iii) $2y^2 + z^2 = 4$ Cylinder with x – axis

(iv) $x^2 + z^2 = 2y^2$ Cone with y – axis

---4-marks

Math. 2 Mid-Term Exam – Spring 2016 Time: 1 Hour

[1]Find **y`** from the following:

(a) $y = 2x^3 + 3^x + 3^{x^2}$ (b) $y = \log x + \sin^{-1} x$

(c) $y = (\cosh x)^{-1} + \ln(1 + \cosh x)$ (d) $y = \tanh^{-1} x + \tan^{-1} x^2$

(e) $y = t.\ln t\,, \quad x = t.\operatorname{sech} t$ (f) $y = x^y + \ln x$

[2]Find the integrals:

(a) $(x^3 + 3^x + \cos 4x)dx$ (b) $\int(\frac{x}{1+x^2} + \frac{2x}{\sqrt{1+x^2}})dx$ (c) $\int(x + \frac{1}{x})^2\, dx$

[3]Find the integrals:

(a) $\int \sin^{-1} x\, dx$ (b) $\int \frac{x}{x^2-3x-4} dx$ (c) $\int sin^4 x\, dx$

[4]Prove that: $\operatorname{sech}^{-1} x = \ln \frac{1+\sqrt{1-x^2}}{x}$

Quiz 1– Math 2

[1]Find y` where:

(a) $y = 3x^3 + 2\cosh x$ (b) $y = 3^x + \tanh x^3$

(c) $y = \tan^{-1}x + (\operatorname{sech} x)^3$ (d) $y = \sin^{-1} x \,.\ln x$

(e) $y = t.\sinh t\,,\ x = t + \ln t$

[2]Find the integrals:

(a) $\int(2x + 2^x + x^2)\,dx$ (b) $\int(\frac{1}{x} - \frac{2}{x-4} + \sin 2x)dx$

(c) $\int(\cosh 3x + \sinh 2x)dx$ (d) $\int sin^{-1}2x\,dx$

Quiz 2 – Math 2

[1]Find **y`** from the following:

(a) $y = 2x^3 + 5^x + 2^{x^3}$ (b) $y = \log x + \sinh^{-1} 2x$

(c) $y = (\cosh x)^{-1} + \ln x$ (d) $y = \sin^{-1} x + \tan^{-1}x^2$

(e) $y = t.\sec t\,,\ x = t.\operatorname{sech} t$ (f) $y^3 = y^x + \ln x$

[2]Find the integrals:

(a) $(\frac{1}{x^4} + 3^x + \cosh 4x)dx$ (b) $\int(\frac{6x}{1+x^2} - \frac{x}{\sqrt{1+x^2}})dx$ (c) $\int(x + \sqrt{x})^2\,dx$

[3]Find the integrals:

(a) $\int \sinh^{-1} 2x\,dx$ (b) $\int \frac{x}{x^2-6x+8}\,dx$ (c) $\int x.\log x\,dx$

[4]Prove that: $\operatorname{sech}^{-1} x = \ln\frac{1+\sqrt{1-x^2}}{x}$